TOWARD A THEORY OF THE SIMULTANEOUS FILTRATION OF IMMISCIBLE LIQUIDS

Yu. A. Buevich and U. M. Mambetov

UDC 532.5

A model and a new formulation are proposed for problems concerning the flow of immiscible fluids in a porous body with allowance for their motion between pores of different sizes. Simple examples are examined.

The simultaneous motion of immiscible liquids in a porous medium is traditionally studied by introducing relative phase permeabilities, dependent on the saturation of one of the liquids, into the Darcy equation. Such an approach has in particular produced the familiar Bakli—Leverette equation [1, 2]. Results which in a number of cases agree quite well with observations have been obtained from the application of such a model to problems involving the displacement of one liquid by another, capillary impregnation, etc. However, the model is actually based on assumptions that are in the nature of postulates which do not always reflect the reality of the given situation. There are questions as to both the degree to which these assumptions are valid and the size of the errors introduced by their use in practical calculations.

First of all, it is assumed that the pressure of each liquid is the same in all of the pores of any small physical volume containing two immiscible liquids, regardless of the sizes of the pores. It is further assumed that the relative permeabilities and the capillary pressure are single-valued functions of saturation. The first of these assumptions corresponds to the hypothesis that processes involving flow between pores of different sizes proceed to completion. The second assumption corresponds to the hypothesis that one of the contacting liquids continuously fills the fine pores while the other fills the coarse pores. Thus, the pressure difference between these liquids, identifiable with the capillary pressure, is determined by the linear scale of the pores in which the liquids come into contact. Otherwise, the dependence of capillary pressure and the relative phase permeabilities on saturation could not be single-valued. These assumptions obviously correspond to a quasisteady approximation when the time scale of the filtration process is much greater than the relaxation time characterizing the dynamics of the fluids' redistribution among the pores.

The latter is far from always the case. For example, if a nonwetting fluid which completely fills a body at the initial moment of time is displaced by a flow of a wetting fluid, the initial displacement occurs not through the fine pores (as it would under equilibrium conditions, due to the wetting fluid's greater affinity for the solid surface) but through the coarse pores (whose permeability is greater). In particular, if the body contains pores differing greatly in structure, size, and permeability, then the opposite relationship usually prevails between the above-indicated time characteristics. Examples of this are filtration flows in cracked-porous media [3-6] and processes involved in the penetration of moisture into soil [7]. In both cases, flows occurring between pores of different sizes play a fundamental role, causing various relaxation phenomena to take place.

An analysis of the internal flow of fluids in the pore space of actual porous bodies requires a detailed knowledge of the topological structure of this space. Such knowledge is currently almost nonexistent. The known granular, capillary, network [1, 6], and fractal [8] models are of a preliminary nature and are in any case inadequate for describing the most important characteristics of porous media. Thus, in the present study we will use a simple phenomenological model describing a porous body as a set of coexisting continua, each of which models a connected system of pores with sizes in a certain interval. Exchange of fluids takes place between the continua. Such a model is a natural generalization of the representation used [3-7].

A. M. Gor'kii Ural State University, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 60, No. 1, pp. 98-107, January, 1991. Original article submitted February 21, 1990.

Let the linear scale ("radius") of the pores change within the interval (a_{min}, a_{max}). We subdivide this interval into N segments of the length δa_j, with the boundaries $a_j = a_{j-1} + \delta a_j$; $a_0 = a_{min}$, $a_N = a_{max}$. We will assume that the pores with the scales in each of these segments form coupled systems, making possible continuous flow through the pores of each of these systems. Such an assumption obviously places a lower limit on the value of δa_j. We will use $\delta\varepsilon_j$ to denote the specific volume of the pores with the dimensions corresponding to the jth segment, while differential permeability — characterizing flow through these pores — will be designated as δk_j. Given sufficiently small δa_j, the following representations are possible

$$\delta\varepsilon_j \approx \left.\frac{d\varepsilon}{da}\right|_{aj} \delta a_j, \quad \delta k_j \approx \left.\frac{dk}{da}\right|_{aj} \delta a_j, \tag{1}$$

where $\varepsilon(a)$ and $k(a)$ are the volume of all pores with scales smaller than a and the permeability associated with motion along such pores. At $a = a_{max}$, these values coincide with the porosity ε and permeability k of the body as a whole.

We will regard pores with scales in the intervals (a_{j-1}, a_j) as interdependent continua. The rate of filtration of a fluid due to its flow through the jth continuum can be determined in accordance with Darcy's law and (1) in the form

$$\delta \mathbf{v}_j = -\frac{\delta k_j}{\mu}(\nabla p_j - \rho \mathbf{g}) \approx -\frac{1}{\mu}\left.\frac{dk}{da}\right|_{aj}(\nabla p_j - \rho \mathbf{g})\,\delta a_j, \tag{2}$$

where p_j is the mean pressure of the liquid in the pores of the size being considered.

We will assume that the liquid is incompressible. Then the material balance equation in the jth continuum can be written in the form

$$\operatorname{div} \delta \mathbf{v}_j = -(q_j^+ - q_j^-), \tag{3}$$

where q_j^+ and q_j^- are the bulk flows in the pores of the jth continuum from other pores and from the pores of this continuum into these other pores, respectively, when calculated per unit volume of the porous body. Under actual situations, a fluid may with a certain probability fall within the investigated range of pore dimensions out of all other possible dimensions; the same situation prevails with regard to the flow of the fluid from the pores. Thus, the quantities q_j^+ and q_j^- actually describe the mean volume of the pores of an isolated jth continuum along with all other remaining pores. The quantities are therefore certain functionals of the pore scale. Here, pursuing the model goals, we assume for the sake of simplification that transport of the fluid between the pores occurs mainly in succession through pores with intermediate linear scales. This proposition obviously imposes the main limitation on the theory being developed here. Then the difference $q_j^+ - q_j^-$ can be understood as $q_{j+1} - q_j$, where q_{j+1} is the flow, referred to a unit volume of the body, from the (j + 1)th continuum to the jth continuum. If we introduce a continuous function $q(a)$ and use the smallness of δa_j, then by analogy with (1) we have

$$q_{j+1} - q_j \approx (\partial q/\partial a)_{a_j}\,\delta a_j. \tag{4}$$

The quantity q_{j+1} may depend on the viscosity of the liquid, the pressure difference $p_{j+1} - p_j$ in the corresponding continua, differential permeability δk_{j+1}, and the scale a_{j+1}. It follows from dimensional theory that

$$q_j = \frac{\alpha_j}{\mu a_j^2}\,\delta k_j (p_j - p_{j-1}) \tag{5}$$

and it then follows from (1)-(3) that:

$$\nabla\left[\left.\frac{dk}{da}\right|_{a_j}(\nabla p_j - \rho \mathbf{g})\right]\delta a_j = \left(\frac{\alpha\delta a}{a^2}\frac{dk}{da}\right)_{a_{j+1}}(p_{j+1} - p_j) - \left(\frac{\alpha\delta a}{a^2}\frac{dk}{da}\right)_{a_j}(p_j - p_{j-1}), \tag{6}$$

where α_j is a certain dimensionless coefficient which is dependent on a_j and which to a certain extent describes the possibility of the penetration of liquid form one group of pores into another, minus the group of pores in the intermediate scale. Actually, (6) constitutes a system of N differential equations in partial derivatives for N unknown pressures p_j. In the particular case when only two continua are introduced, this system is transformed into the problems examined in [3-7].

Assuming δa_j to be small, taking (4) into account, and introducing p as a continuous function of a and its other arguments, we can employ a standard procedure to change over from finite-difference equations (6) to a single differential equation. As a result, we obtain the equation

$$\nabla\left[\frac{dk}{da}(\nabla p-\rho g)\right]-\frac{\partial}{\partial a}\left(\beta\frac{dk}{da}\frac{\partial p}{\partial a}\right)=0, \qquad (7)$$

$$\beta(a_j)=\alpha_j a_j^{-2}\delta a_j\delta a_{j-1},$$

which is of the hyperbolic type.

For many porous bodies, the topological properties of the pore space are approximately the same for different scale levels. This fact has in particular stimulated the formulation of the principle of statistical structural similitude. The use of this principle in defining porosity within the framework of the model of random fractals proposed in [8] produced quite satisfactory results. Similitude of the structure means that the function $\beta(a)$ determined in (7) can be regarded as a constant which, by definition, should be much less than unity. If the field of the external body forces is independent of the coordinates and if the porous body is macroscopically uniform, then (7) leads to the equation

$$\frac{dk}{da}\Delta p-\beta\frac{\partial}{\partial a}\left(\frac{dk}{da}\frac{\partial p}{\partial a}\right)=0, \qquad (8)$$

which should be satisfied for all immiscible fluids simultaneously filling the porous body. We emphasize that the number of such fluids may be greater than the number two usually examined in the theory [1, 2].

The boundary between fluids in contact in pores of a certain scale is a certain hypersurface in four-dimensional space $\{\mathbf{r}, a\}$ and is dependent on time as a parameter. We will assign the equation of such a hypersurface Γ in the form $F(\mathbf{r}, a, t) = 0$ and will examine the boundary conditions which should be imposed on it. First of all, the pressure difference between the two sides of the hypersurface is the capillary pressure for the pores of the boundary scale, i.e.,

$$p|_{n=+0}-p|_{n=-0}=p_c=\sigma/a,\quad F(\mathbf{r},\ a,\ t)=0, \qquad (9)$$

where n is a coordinate which is normal to Γ and has its origin on Γ. It is directed toward the side of the four-dimensional region occupied by the less-wetting fluid. The quantity σ is proportional to the surface tension at the boundary of the fluids being examined. This surface tension is dependent on the structure of the pore space.

A kinematic condition connecting the displacement of this hypersurface with the mean fluid velocity in the pores in the space $\{\mathbf{r}, a\}$ must then be imposed on Γ. The mean velocity of the fluid in the pores is determined by the four-dimensional vector $\mathbf{u}^*$. The first three components of this vector, corresponding to the three-dimensional physical space, are determined by dividing the rate of filtration $\delta\mathbf{v}_j$ from (2) by the differential porosity $\delta\varepsilon_j$ from (1):

$$\mathbf{u}=\frac{\delta\mathbf{v}_j}{\delta\varepsilon_j}=-\frac{1}{\mu}\frac{dk}{d\varepsilon}(\nabla p-\rho g), \qquad (10)$$

while the fourth component is found by dividing the flow q from (5) by $\delta\varepsilon_j$, i.e.,

$$u_a=\frac{\partial a}{\partial t}=-\frac{\beta}{\mu}\frac{dk}{d\varepsilon}\frac{\partial p}{\partial a}, \qquad (11)$$

where $a(\mathbf{r}, t)$ represents the root of the equation $F(\mathbf{r}, a, t) = 0$.

We now proceed similarly to derive the boundary condition for a movable solid boundary in the mechanics of an ideal fluid [9]. The normal rate of displacement of the hypersurface Γ in the space $\{\mathbf{r}, a\}$, which can be expressed as

$$U_n=-\frac{\partial F/\partial t}{|\nabla^* F|},\quad \nabla^*=\left\{\nabla,\ \frac{\partial}{\partial a}\right\},$$

should coincide with the normal velocity of the fluid in this space on Γ. This velocity is equal to the scalar product of

$\mathbf{u}^*$ and the unit normal vector $\nabla^* F|\nabla^* F|^{-1}$. As a result, with allowance for (10) and (11) we obtain the condition

$$\frac{\partial F}{\partial t} = \frac{1}{\mu}\frac{dk}{d\varepsilon}\left[(\nabla p - \rho \mathbf{g})\frac{\partial F}{\partial \mathbf{r}} + \beta\frac{\partial p}{\partial a}\frac{\partial F}{\partial a}\right], \quad F(\mathbf{r}, a, t) = 0, \tag{12}$$

which determines the function $F(\mathbf{r}, a, t)$. On the sections of Γ corresponding to the values of a, i.e., the hyperplanes $a = a_{\min}$ and $a = a_{\max}$, the flow $q(a)$ should vanish. Following from this are the trivial conditions

$$\partial p/\partial a = 0, \quad a = a_{\min}, \quad a = a_{\max}. \tag{13}$$

Conditions (9), (12), and (13) must be assigned on the boundaries between all regions in $\{\mathbf{r}, a\}$ that are continuously filled by different immiscible fluids. These conditions must also be satisfied for fluids on both sides of each boundary. In particular, (12) is simultaneously valid for both contacting fluids. It follows from this that the quantities in the right side of (12) are continuous with the crossing of the bounding hypersurface. Thus, we have Eqs. (8) for all of the fluids, conditions (9) and (12) on the boundaries of the regions that these fluids fill, conditions (13), and the initial and boundary conditions dictated by the features of the filtration process being examined.

Let a certain fluid continuously fill all pores with the scale in the interval (a_1, a_2), where $a_k(\mathbf{r}, t)$ are determined as the solutions of the equations $F_k(\mathbf{r}, a, t) = 0$ of the corresponding hypersurfaces. If the relaxation time of the processes involving flow between pores of different sizes is much shorter than the time scale of the filtration flow, then the pressure is slightly dependent on a. Integrating (8) over da from a_1 to a_2 and allowing for (2), we approximately obtain

$$K(a_1, a_2)\Delta p \approx \beta\frac{dk}{da}\frac{\partial p}{\partial a}\Big|_{a_1}^{a_2} = -\mu\left(u_a\frac{d\varepsilon}{da}\right)_{a_1}^{a_2} = \mu\frac{\partial a_2}{\partial t}\left(\frac{d\varepsilon}{da}\right)_{a_2} - \mu\frac{\partial a_1}{\partial t}\left(\frac{d\varepsilon}{da}\right)_{a_1} = -\mu\varepsilon\frac{\partial S(a_1, a_2)}{\partial t}, \tag{14}$$

where we have introduced permeability and saturation

$$K(a_1, a_2) = k(a_2) - k(a_1), \quad \varepsilon S(a_1, a_2) = \varepsilon(a_2) - \varepsilon(a_1). \tag{15}$$

It is evident from this that, in the general case, K is not a single-valued function of saturation S — the hypothesis usually employed notwithstanding. The usual hypothesis is valid only in the case when one of the boundary conditions of the linear scale is fixed ($a_1 = a_{\min}$ or $a_2 = a_{\max}$).

If two fluids respectively occupy pores with $a < a_*$ and $a > a_*$ (subscripts 1 and 2), then from (9), (14), and (15) we have

$$K_j(a_*)\Delta p_j = -\mu\varepsilon\partial S_j(a_*)/\partial t, \quad p_2 - p_1 = \pm\sigma/a_*,$$
$$K_1(a_*) = K(a_{\min}, a_*), \quad K_2(a_*) = K(a_*, a_{\max}), \quad S_1 + S_2 = 1, \tag{16}$$

which corresponds to the standard model of two-phase filtration [1, 2]. Given certain assumptions, the standard Bakli–Leverette equation can be obtained directly from (16) or from (12) in the unidimensional case. The single-valued dependences of relative phase permeability and capillary pressure on saturation S_1 (or S_2) which are used in this model are obtained after exclusion of the parameter a_* from (16).

To perform specific calculations, it is necessary to assign the functions $\varepsilon(a)$ and $k(a)$. In accordance with the Kozeny theory and other models of porous media and with allowance for the above-employed hypothesis on structural similitude, we take

$$\frac{dk}{da} = \gamma a^2\frac{d\varepsilon}{da}, \tag{17}$$

where γ is a dimensionless coefficient which is independent of a. We will determine differential porosity by means of a power relation that has been empirically substantiated for many porous media (limestones, sandstones, clayey materials, etc.) and which follows theoretically from the model of random fractals [8]

$$\frac{d\varepsilon}{da}=\frac{C}{a^n},\quad C=\varepsilon(n-1)\left(\frac{1}{a_{\min}^{n-1}}-\frac{1}{a_{\max}^{n-1}}\right)^{-1} \tag{18}$$

(for simplicity, we ignore the sharp maximum in the region of the smallest pores). In this case, the functions $\varepsilon(a)$ and $k(a)$ are represented in the form:

$$\varepsilon(a)=\varepsilon\left(\frac{a_{\max}}{a}\right)^{n-1}\frac{a^{n-1}-a_{\min}^{n-1}}{a_{\max}^{n-1}-a_{\min}^{n-1}},$$
$$k(a)=\gamma\varepsilon\frac{n-1}{3-n}\frac{(a_{\min}a_{\max})^{n-1}}{a_{\max}^{n-1}-a_{\min}^{n-1}}(a^{3-n}-a_{\min}^{3-n}). \tag{19}$$

The character of Eqs. (19) is quite different with different n; usually, $1 < n < 2$. These formulas are simplified considerably when $a_{\max} \gg a_{\min}$.

Let us examine some simple examples involving the solution of the proposed equations. Let a porous body contain a wetting fluid which fills pores with scales in the interval (a_1, a_2). For simplicity, we will ignore the presence of any other substance in the other pores. It then follows from (8) and (9) that

$$p=-\frac{\sigma}{a_1}+\left(\frac{\sigma}{a_1}-\frac{\sigma}{a_2}\right)\left(\int\limits_{a_1}^{a_2}\frac{da}{dk/da}\right)^{-1}\int\limits_{a_1}^{a}\frac{da}{dk/da}. \tag{20}$$

Since the equations of the boundary hypersurfaces in the given case have the form $a-a_j(t)=0$, we find from (12) and (20) that

$$\frac{da_j}{dt}=-\frac{\beta}{\mu}\left(\frac{d\varepsilon}{da}\right)_{a_j}^{-1}\left(\int\limits_{a_1}^{a_2}\frac{da}{dk/da}\right)^{-1}\left(\frac{\sigma}{a_1}-\frac{\sigma}{a_2}\right),\quad j=1,\ 2. \tag{21}$$

Inserting (17) into (18), we obtain the system of equations

$$\frac{da_j}{dt}=\frac{(n-1)\beta\gamma\sigma}{\mu}\frac{a_j^n}{a_2^{n-1}-a_1^{n-1}}\left(\frac{1}{a_1}-\frac{1}{a_2}\right), \tag{22}$$

which are simplified considerably at $a_1 \ll a_2$, when the solution of the system is as follows:

$$\frac{1}{a_1^{n-1}}-\frac{1}{a_2^{n-1}}=\frac{R}{a_{10}^{n-1}},\quad R=1-\left(\frac{a_{10}}{a_{20}}\right)^{n-1}\approx 1,$$
$$\int\limits_{a_1/a_{10}}^{1}\frac{dz}{1-Rz^{n-1}}=\frac{t}{\tau},\quad \tau=\frac{\mu}{(n-1)\beta\gamma\sigma a_{10}},\quad a_{j0}=a_j(0). \tag{23}$$

Given realistic values for the physical parameters, the relaxation time of the process of flow of the wetting fluid into finer pores may prove to be substantial if the parameter β is sufficiently small. Similar results for a nonwetting fluid are obtained from the above relations with a change in the sign of σ: in this case, the coarser pores are filled. The interpore flow process is slowed considerably and eventually practically ceases if there is a gas in the pores with $a < a_1$ and $a > a_2$. Within the framework of the model being examined, this gas is completely fixed. Its pressure changes during compression in the fine pores and during expansion in the coarse pores in accordance with the equation of state.

Now let us examine the problem of the unidimensional displacement of a gas by a drop liquid in the case when the hydraulic resistance to the flow of the liquid can be ignored as a first approximation. If the parameter β is very small compared to unity, then it is natural to seek the solution of the boundary-value problem for Eq. (8) in the form of a series in powers of β. If the porous body is completely filled with gas at the initial moment of time, then the equation of the bounding hypersurface can be written in the form $x-h(a, t)=0$, $h(a, 0)=0$. Then from (8)-(9) we obtain the following as the principal term of the series for the pressure, which is proportional to $\beta^0 = 1$

$$p^{(0)}(x,\ a,\ t) = P(t) - \left[P(t) \pm \frac{\sigma}{a}\right]\frac{x}{h(a,\ t)}, \tag{24}$$

where the top and bottom signs pertain to the wetting and nonwetting fluids, respectively. With allowance for (24), we find the following from (12) for small β

$$\frac{\partial h}{\partial t} = \frac{1}{\mu}\frac{dk}{d\varepsilon}\left(\frac{P(t) \pm \sigma/a}{h(a,\ t)} + \rho g_x\right), \tag{25}$$

the solution of which has the below form in the special case when the pressure $P(t) = p_0$ is assigned on the boundary $x = 0$

$$h - h_* \ln\left(1 + \frac{h}{h_*}\right) = \frac{\rho}{\mu}\frac{dk}{d\varepsilon} g_x t,\ h_* = \frac{p_0 \pm \sigma/a}{\rho g_x} \tag{26}$$

(for the sake of determinateness, we assume that g_x is positive). The asymptotes below follow from (26) at small and large h

$$h \approx \left[\frac{2}{\mu}\left(p_0 \pm \frac{\sigma}{a}\right)\frac{dk}{d\varepsilon} t\right]^{1/2},\quad h \ll h_*,$$

$$h \approx \frac{\rho}{\mu}\frac{dk}{d\varepsilon} g_x t,\quad h \gg h_*.$$

If Eq. (17) is valid, then at $p_0 > 0$ the quantity $h(a, t)$ is a monotonically increasing function of both arguments. Meanwhile, the wetting fluid flows through all pores simultaneously, but the nonwetting fluid is capable of penetrating only those pores with $a > \sigma/p_0$. At $p_0 < 0$, only flow of the wetting fluid in pores with the scales $a < \sigma/|p_0|$ is possible; in this case, $h(a, t)$ has a maximum as a function of a. These features have much in common with the results obtained in [7].

In order to evaluate the effect of interpore flow on the form of the bounding hypersurface, it is necessary to examine the subsequent terms of the expansion of p in powers of β and to account for the term with β in (12). If we do this, then instead of (25) we obtain a first-order partial differential equation. The expansion turns out to be nonuniformly valid with respect to time. However, it follows from an analysis of the process for short periods of time that, in the case of a wetting fluid, exchange between pores helps weaken the dependence of h on a. In the case of a nonwetting fluid, it helps strengthen this dependence compared to (26).

If we assign the total volume flow rate of the fluid $\varepsilon Q(t)$ at the inlet ($x = 0$) instead of pressure $P(t)$, then, as before, Eq. (24) is approximately valid for small β. However, in this case $P(t)$ needs to be determined from the equation

$$\varepsilon Q(t) = \frac{1}{\mu}\int_{a_{min}}^{a_{max}} \frac{dk}{da}\left(\frac{P(t) \pm \sigma/a}{h(a,\ t)} + \rho g_x\right) da. \tag{27}$$

The asymptotic displacement regime attained at $t \to \infty$ is of interest. We will assume that in a process with a specified constant flow rate, the form of the bounding hypersurface, moving along x at a constant velocity, ceases to depend on time. We then have

$$h(a,\ t) \sim Qt + H(a),\ H(a_{\min}) = 0,\ P(t) \sim Pt,$$

so that at $t \to \infty$ the equation of the hypersurface $x - Qt - H(a) = 0$. In addition:

$$\frac{P(t) \pm \sigma/a}{h(a,\ t)} \sim \frac{P}{Q}.$$

Ignoring the external body forces for the sake of simplification, we use (27) to obtain $P = \varepsilon\mu Q^2/k$. Taking this into account, we use (12) to obtain the following instead of (25)

$$\left(\frac{dH}{da}\right)^2 \pm \frac{k\sigma}{\varepsilon\mu a^2 Q}\frac{dH}{da} + \frac{1}{\beta}\left(1 - \frac{k}{\varepsilon dk/d\varepsilon}\right) = 0, \tag{28}$$

from which we then obtain

$$\frac{dH}{da} = \frac{f}{z}\left\{\mp 1 + \left[1 + \frac{z^2}{\beta f^2}\left(\frac{2a^2 f}{dk/d\varepsilon} - 1\right)\right]^{1/2}\right\}, \tag{29}$$
$$z = \mu Q/\sigma,\ f = k/2\varepsilon a^2.$$

Thus, the postulated asymptotic regime can exist only in the case when the expression in the square brackets in (29) is positive for any a in the interval $(a_{\min}, a_{\max})$. Assuming for the sake of determinateness that Eqs. (17)-(19) are valid with $1 < n < 3$, we see that this holds at

$$f = \frac{\gamma}{2}\frac{n-1}{3-n}\frac{b^{3-n}-1}{b^2(b^{1-n}-1)} > \frac{\gamma}{2},\quad b = \frac{a}{a_{\min}}, \tag{30}$$

as well as in the case when

$$f < \frac{\gamma}{2},\quad z^2 < \beta f^2\left(1 - \frac{2}{\gamma}f\right)^{-1} \tag{31}$$

for pores of all scales. It is not hard to see that inequality (30) is satisfied for all a at $2 < n < 3$. However, the opposite inequality holds in the case of a shallower relation (18) corresponding to $1 < n < 2$, when the percentage of coarse pores is relatively high. In this case, the above-examined asymptotic regime fails to be established either for the wetting fluid or for the nonwetting fluid if the flow rate Q is high enough, i.e.,

$$Q > Q_* = \frac{\gamma}{2}\left[\frac{\beta(n-1)^2}{2(3-n)(n-2)}\right]^{1/2}\frac{\sigma}{\mu},\quad 1 < n < 2. \tag{32}$$

When the above inequalities are satisfied, the flow regimes becomes essentially unsteady: the form of the bounding hypersurface changes even over long periods of time. If the fluid being displaced is not a gas and if this is necessary to consider hydraulic resistance to its flow, then oscillatory displacement regimes corresponding to a particular mode of "flooding" may also evidently be established. In these regimes, the presence of a sufficient number of coarse and relatively permeable pores leads to "breakthrough" of the displacing fluid to them. As a result, remaining behind the fluid front are closed volumes in which part of the pore space is occupied by fluid which has not yet been able to escape.

If condition (30) or condition (31) are satisfied, then the function $H(a)$ is obtained from (29) by a simple formula of integration. The function decreases monotonically with an increase in a for a wetting fluid and increases with a for a nonwetting fluid. Due to a shortage of space, the relations $H(a)$ and the saturation distributions $S(x)$ which result from these relations are not presented here. In contrast to the Bakli—Leverette theory, the functions $S(x)$ do not have a discontinuity on the displacement front.

Similar results can easily be obtained for flow regimes in which pressure is assigned at the inlet $x = 0$ if we consider the effect of external body forces. In this case, the quantity ρg_x plays the role of P/Q, while the value of Q at $t \to \infty$ is determined from (27).

One important conclusion which follows from the above examples is the strong effect of pore-size distribution on both the quantitative and the qualitative characteristics of the displacement regimes. Here, a particularly important role is played by coarse pores. The latter ensure rapid flow of the displacing fluid accompanied by disconnection of the regions filled by the fluid being displaced. It can be expected that such disconnection will be even more important for materials possessing dual porosity and having a bimodal pore distribution. Cloddy soils are an example of this class of materials.

The main shortcoming of the model developed above has to do with the assumption that fluid flows in succession through pores with intermediate scales during its movement between pores of different sizes. This precludes direct exchange between different-size pores, which in turn ultimately leads to the neglect of important hysteresis effects. In particular, it leads to equality of the sum of the relative phase permeabilities in (16) to unity. The use of the above-mentioned assumption also means that it is impossible for gas to be exchanged between fine and coarse pores when the intermediate-scale pores contain a drop liquid. Thus, the most important goal of future attempts to improve the theory is to avoid having to use this assumption and allow for the possibility of the movement of fluid between same-size pores through pores of a different size. Such a goal can be attained in part through the further development of network models [6, 10-12] of porous materials.

NOTATION

Here a is the linear scale of pores; b, dimensionless scale in (30); C, coefficient in (18); F, function determining the boundary hypersurface; f, function determined in (29); g, acceleration of the external mass field; H, h, functions entering into the determination of F; K, phase permeability; n, exponent in the scale distribution of porosity; P, coefficient in the determination of the boundary pressure $P(t)$; p, pressure; R, parameter in (23); $\mathbf{r}$, position vector; εQ, flow rate of fluid; q, flow along the axis of the pore scales; S, saturation; t, time; $\mathbf{u}$, u_a, velocity determined in (10) and (11); x, coordinate; z, dimensionless flow rate; α, β, coefficients in (5) and (7); γ, parameter in (17); ε, porosity; μm viscosity; ρ, density; σ, quantity proportional to the surface tension, entering into the determination of capillary pressure; τ, relaxation time determined in (23).

LITERATURE CITED

1. A. E. Sheidegger, Physics of Flow through Porous Media [Russian translation], Moscow (1960).
2. R. Collins, Flows of Liquids through Porous Materials [Russian translation], Moscow (1964).
3. G. I. Barenblatt, Yu. N. Zheltov, and I. N. Kochina, Prikl. Mat. Mekh., **24**, No. 5, 852-864 (1960).
4. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, Theory of the Unsteady Filtration of Liquids and Gases [in Russian], Moscow (1972).
5. Yu. A. Buevich, Inzh.-Fiz. Zh., **46**, No. 4, 593-600 (1984).
6. E. S. Romm, Structural Models of the Pore Space of Rocks, Leningrad (1985).
7. Yu. A. Buevich and U. M. Mambetov, Inzh.-Fiz. Zh., **56**, No. 6, 953-960 (1989).
8. R. R. Nigmatulin and N. N. Sutugin, Inzh.-Fiz. Zh., **57**, No. 2, 291-298 (1989).
9. M. A. Lavrent'ev and B. V. Shabat, Problems of Hydrodynamics and Their Mathematical Models [in Russian], Moscow (1977).
10. I. Fatt, Trans. AIME, **207**, No. 2, 160-181 (1956).
11. V. S. Markin, Dokl. Akad. Nauk SSSR, **15**, No. 3, 620-623 (1963).
12. V. M. Entov, A. Ya. Fel'dman, and E. Chen-Sin, Programmirovanie, No. 3, 67-74 (1975).